IS EQUAL TO

Logarithms 1

Solve the following problems.

1. Simplify $\log_2 16$.
2. Express $\log_3 81$ in terms of $\log 3$ and $\log 81$.
3. Solve the equation $2\log(x+3) = \log 24$ for x.
4. Solve $10^{2x} = 1000$.
5. Solve $\ln(x+1) = 3$ for x.
6. The half-life of a radioactive substance is given by $N(t) = N_0 e^{-kt}$, where N_0 is the initial amount. If the half-life is 5 years, find k.
7. Solve the equation $3e^{2x+1} = 17$, giving your answer to three significant numbers.
8. Solve the equation $7^{x-2} = 49$.
9. Solve $2^{2x} + 2^{x+1} - 3 = 0$.
10. The Richter scale measures the magnitude of earthquakes using the formula $M = \log \frac{A}{A_0}$. If an earthquake has an amplitude of 10^5 micrometers, calculate the magnitude of the earthquake.
11. Evaluate the following:

    (a) $\log_3 27$ (b) $\log_{12} 1$ (c) $\log_5 5$ (d) $\log_3 \frac{1}{3}$

12. Simplify the following given $b > 0$:

    (a) $\log_b b^4$ (b) $\log_b b^{1/2}$ (c) $\log_{b^{1/2}} b^3$ (d) $\log_b b^2 - \log_b b^{1/2}$

13. Given $x = \log a$, $y = \log b$, and $z = \log c$, express the following in terms of x, y, and z:

    (a) $\log b^7$ (b) $\log(\frac{ab^2}{c})$ (c) $\log(\frac{100}{bc^5})$ (d) $\log a^3 - 2\log ab^2$

14. Solve for x, giving your answers correct to 3 significant figures.

    (a) $3\times 4^x = 90$ (b) $6\times 7^{x+1} = 1.2$ (c) $3^{2x} = 4^{x-1}$ (d) $3\times 2^{3x} = 7\times 3^{3x-2}$

15. Solve the following equations, giving your answers in terms of natural logarithms.

(a) $2 \times 3^x = 5$ (b) $4e^x = 1$ (c) $2^{2x} = 5$ (d) $2^{3x+1} = 10$

16. Find the exact solution(s) of each equation.

(a) $4x - 5 \times 2^x + 6 = 0$

(b) $9^x - 8 \times 3^x = 9$

(c) $e^{2x} + 16e^x = 80$

(d) $25^x - 15 \times 5^x + 50 = 0$

17. Solve the following equations:

(a) $3^{2x} - 5 \times 3^x + 4 = 0$

(b) $2^{2x} + 2^{x+1} - 3 = 0$

(c) $5^{2x} - 6 \times 5^x + 5 = 0$

18. (a) The population of a town doubles every 5 years. If the initial population is 1000, write an exponential model for the population $P(t)$ at time t years.

(b) Find the population after 12 years.

(c) How long will it take for the population to reach 8000?

19. Solve the following for x:

(a) $\log_5\left(\frac{3+x}{x-2}\right) = 2$

(b) $\log_3(4x + 1) = 4$

(c) $\log_4 x - \log_4(x - 4) = 2$

(d) $\log_9 x - 3\log_9\left(\frac{2}{x}\right) = 2$

20. The temperature of a hot object cools according to Newton's Law of Cooling, which states that the rate of change of the temperature is proportional to the difference between the temperature of the object and the ambient temperature. The temperature at time t is given by the exponential decay formula:

$$T(t) = T_a + (T_0 - T_a)e^{-kt}$$

where: - $T_0 = 100°C$ is the initial temperature of the object, - $T_a = 20°C$ is the ambient temperature, - $k = 0.03$ is the cooling constant, - t is the time in hours.

(a) Calculate the temperature of the object after 5 hours.

(b) After how many hours will the object's temperature drop to 40°C?

(c) If the object had started at 150°C instead of 100°C, how long would it take for the object's temperature to reach 60°C?